major character and admit a generalization of relations both of the first and of the second types. In particular, we can take into account creep, nonorthogonality of the slip lines, dilatational effects [10], and effects of internal friction which have importance for soils and rocks.

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# FORMING OF FIBROUS LIGHTGUIDES WITH A SMALL AZIMUTHAL ASYMMETRY OF THE BILLET


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One possible type of fibrous lightguide is a transparent microcapillary. Small losses with the propagation of light along a lightguide are possible if its transverse cross section is sufficiently close to a concentric round ring and is constant over the length of a fiber. From a physical point of view, the process of the forming of a lightguide can be represented as the flow of an incompressible Newtonian liquid with a variable viscosity (some polymers are not Newtonian liquids and are therefore not discussed here).

Article [1] discusses the pulling of a microcapillary from a billet, i.e., a solid hollow cylinder of given dimensions. The billet and all the external conditions under which the pulling was done were assumed to be axisymmetric, as a result of which the microcapillary pulled was also axisymmetric with a round cross section. In [1] equations for the form of the jet (the transition from the billet to the microcapillary) were obtained and the dependence of the dimensions of the microcapillary on the parameters of the process was found. We discuss below the pulling of a microcapillary from a billet, taking account of the small real nonaxisymmetric character of the latter; the degree of nonaxisymmetry of the microcapillary is found and its dependence on the parameters of the process is investigated.

§1. In all aspects, except for the assumption of the nonaxisymmetry of the process, the statement of the problem is the same as in [1]: the temperature distribution is assumed to be given; in all cross sections, the thickness of the wall of the billet and the jet is assumed to be small in comparison with its radius; by virtue of the thinness of the wall, the temperature is assumed to be identical at all points of the transverse cross section of the jet and to depend only on the longitudinal coordinate z; the viscosity is a known function of the tem-


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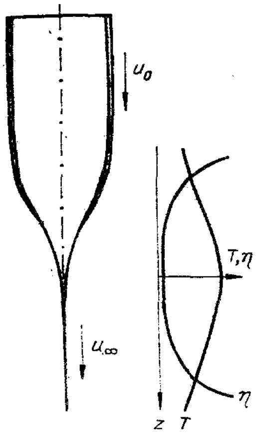

Fig. 1

perature and its distribution is described by a given function $\eta(z)$ (Fig. 1, where on the right there are given typical profiles of the temperature of the furnace and the viscosity of the material); $\eta(z)\to\infty$ as $z\to\pm\infty$; the rate of feeding of the billet $u_0$ and the rate of pulling of the microcapillary $u_\infty$ are assumed to be given quantities. Account is taken of the surface tension $\sigma$ and the pressure drop of the air $\Delta p = p_1 - p_2$ between the channel and the external medium; the values of $\sigma$, $p_1$ and $p_2$ are assumed to be identical for all the sections of the jet.

The form of the tranverse cross section of the billet (Fig. 2, where, for clarity, the nonaxisymmetric cross section and the thickness of the wall are greatly enlarged) is regarded as given and is described by two functions of the azimuthal angle $\varphi$: $h_0(\varphi)$, the thickness of the wall, and $\bar{r}_0(\varphi)$, the mean radius. In the expansion of the functions $h_0(\varphi)$ and $\bar{r}_0(\varphi)$ in Fourier series

$$h_0(\varphi) = h_0^{(0)}\left[1+\sum_{n=1}^{\infty}\varkappa_{n,0}\cos n\varphi\right];\quad \bar{r}_0(\varphi) = \bar{r}_0^{(0)}\left[1+\sum_{n=1}^{\infty}\rho_{n,0}\cos n\varphi\right] \tag{1.1}$$

the values of $h_0^{(0)}$ and $\bar{r}_0^{(0)}$ are mean values; $\varkappa_{n,0}$ and $\rho_{n,0}$ are the relative amplitudes of the harmonic of the thickness of the wall and the mean radius.* By a choice of the origin of the system of coordinates, we can always obtain

$$\rho_{1,0} = 0. \tag{1.2}$$

The nonaxisymmetry of the transverse cross section of the billet is assumed to be small:

$$|\varkappa_{n,0}| \ll 1;\ |\rho_{n,0}| \ll 1,\ n = 1, 2, 3, \ldots \tag{1.3}$$

The sought quantity is the form of the transverse cross section of the microcapillary, described by the functions $h_\infty(\varphi)$ and $\bar{r}_\infty(\varphi)$,

$$h_\infty(\varphi) = h_\infty^{(0)}\left[1+\sum_{n=1}^{\infty}\varkappa_{n,\infty}\cos n\varphi\right];\quad \bar{r}_\infty(\varphi) = \bar{r}_\infty^{(0)}\left[1+\sum_{n=1}^{\infty}\rho_{n,\infty}\cos n\varphi\right]. \tag{1.4}$$

By virtue of the axisymmetry of all the external conditions, in the first approximation with respect to small values of (1.3), the mean values of $h_\infty^{(0)}$ and $\bar{r}_\infty^{(0)}$ do not depend on the values of $\varkappa_{n,0}$ and $\rho_{n,0}$ and can be found from a solution of the axisymmetric problem [1], while the relative amplitudes of the n-th harmonic in (1.4) depend on the relative amplitudes only of this n-th harmonic in (1.1):

$$\varkappa_{n,\infty} = A_n\varkappa_{n,0} + B_n\rho_{n,0}; \tag{1.5a}$$

$$\rho_{n,\infty} = C_n\varkappa_{n,0} + D_n\rho_{n,0}. \tag{1.5b}$$

The aim of the work was to determine the coefficients of the transition $A_n$, $B_n$, $C_n$, and $D_n$ and to investigate their dependence on the parameters of the process.

§2. We seek the solution of the problem posed in the form of a small [by virtue of (1.3)] nonaxisymmetric perturbation of the axisymmetric flow of the liquid, found in [1]. We represent the pressure p, the components

*In formula (1.1) the expansion in terms of sin $n\varphi$ is omitted. An analysis of the total solution taking account of sin $n\varphi$ shows that taking them into consideration yields nothing new.

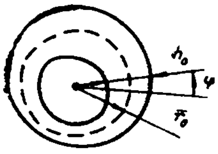


Fig. 2

of the velocity of the flow of liquid $v_r$, $v_\varphi$, $v_z$, the radii of the internal and external boundaries of the jet $r_1(z, \varphi)$, $r_2(z, \varphi)$, and the associated thickness of the wall $h(z, \varphi)$ and mean radius of the jet $\bar r(z, \varphi)$

$$h(z,\ \varphi) = r_2(z,\ \varphi) - r_1(z,\ \varphi);\quad \bar r(z,\ \varphi) = (1/2)[r_1(z,\ \varphi) + r_2(z,\ \varphi)] \tag{2.1}$$

in the form of the sum of quantities of the zero (with respect to powers of $\varkappa_{n,0}$ and $\rho_{n,0}$) and first approximations

$$\begin{gathered} p = p^{(0)} + p^{(1)};\quad v_r = v_r^{(0)} + v_r^{(1)};\quad v_\varphi = 0 + v_\varphi^{(1)};\quad v_z = v_z^{(0)} + v_z^{(1)}; \\ r_1 = r_1^{(0)} + r_1^{(1)};\quad r_2 = r_2^{(0)} + r_2^{(1)};\quad h = h^{(0)} + h^{(1)};\quad \bar r = \bar r^{(0)} + \bar r^{(1)}. \end{gathered} \tag{2.2}$$

Quantities with the superscript 1, depending on $\varphi$, are small in comparison with the corresponding quantities with the superscript 0, which do not depend on the angle $\varphi$ and which describe axisymmetric flow.

The equations of axisymmetric flow, obtained in [1] by simplification of the Navier–Stokes equations, the equation of continuity, and the boundary conditions at the lateral surfaces, on the basis of the assumption of the relative thinness of the wall of the billet and the smallness of the angle of inclination of the jet $\theta$ in the plane $r-z$, have the form

$$h_0^{(0)}/\bar r_0^{(0)} \ll 1; \tag{2.3a}$$

$$\theta \sim \bar r_0^{(0)}/l \ll 1, \tag{2.3b}$$

where $l$ is the unit characteristic dimension along the z axis, having the sense of the length of the heating zone:

$$l/\eta_0 = \int_{-\infty}^{\infty} dz/\eta(z) \tag{2.4}$$

($\eta_0$ is the minimal viscosity). After the introduction of the dimensionless parameters and dimensionless variables

$$U_\infty = \frac{u_\infty}{u_0};\quad w = \ln U_\infty;\quad P = \frac{\Delta p \bar r_0^{(0)} l}{2\eta_0 u_0 h_0^{(0)} w};\quad Q = \frac{\sigma l}{\eta_0 u_0 h_0^{(0)} w}; \tag{2.5}$$

$$s(z) = w\eta_0/l \int_{-\infty}^{z} d\zeta/\eta(\zeta)\qquad (s(-\infty) = 0;\ s(+\infty) = w); \tag{2.6}$$

$$H[s(z)] = h^{(0)}(z)/h_0^{(0)};\quad R[s(z)] = \bar r^{(0)}(z)/\bar r_0^{(0)};\quad U[s(z)] = v_z^{(0)}(z)/u_0 \tag{2.7}$$

the equations and boundary conditions for the dimensionless longitudinal velocity and the mean radius and wall thickness of the jet have the form [1]

$$\begin{cases} dU/ds = \gamma U - QRU/3; \\ dR/ds = PR^3/3 - QR^2/2 - \gamma R/2; \end{cases} \tag{2.8}$$

$$U|_{s=0} = 1;\ R|_{s=0} = 1;\ U|_{s=w} = U_\infty; \tag{2.9}$$

$$H(s) = R^{-1}(s)U^{-1}(s), \tag{2.10}$$

where constant $\gamma$ is determined from the boundary conditions (2.9). Integration of (2.8) gave an expression for the dimensions of the microcapillary $h_\infty^{(0)}$ and $\bar{r}_\infty^{(0)}$:

$$h_\infty^{(0)} = h_0^{(0)} U_\infty^{-1/2} K(P, Q, w); \quad \bar{r}_\infty^{(0)} = \bar{r}_0^{(0)} U_\infty^{-1/2} K^{-1}(P, Q, w). \tag{2.11}$$

In [1] a detailed curve was given for the function K, and, from an analysis of this curve, it was shown that the following are of interest:

$$Q \leqslant 3; \; K > 1. \tag{2.12}$$

§3. In the r-th and $\varphi$-th components of the Navier–Stokes equation, in which, in view of the smallness of the Reynolds number, nonlinear terms are omitted, on the basis of (2.3b) we neglect the term $\partial^2 v_\varphi^{(1)}/\partial r^2$ in comparison with $\partial^2 v_\varphi^{(1)}/\partial z^2$ and the term $\partial^2 v_\varphi^{(1)}/\partial r^2$ in comparison with $\partial^2 v^{(1)}/\partial r^2$. In the equation of continuity, taking into consideration that $v_z^{(1)} \sim v_r^{(1)}$, we neglect the term $\partial v_z^{(1)}/\partial z$ in comparison with $\partial v_r^{(1)}/\partial r$. After these simplifications, the pressure $p^{(1)}$ and the transverse components of the velocity $v_r^{(1)}$ and $v_\varphi^{(1)}$ satisfy the two-dimensional Navier–Stokes equations and the two-dimensional equation of continuity, into which the variable z enters only parametrically:

$$\nabla_t p^{(1)} = \eta(z) \nabla_t^2 \mathbf{v}_t^{(1)}; \quad \nabla_t \cdot \mathbf{v}_t^{(1)} = 0, \tag{3.1}$$

where $\nabla t = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \varphi}\right)$ is the operator of differentiation over the transverse coordinates; $\mathbf{v}_t^{(1)} = (v_r^{(1)}, v_\varphi^{(1)})$. Equations (3.1) reduce to a biharmonic equation for the stream function, solving which we have

$$v_r^{(1)} = \sum_{n=1}^{\infty} \vartheta_n(r) \cos n\varphi \equiv (a_1 r^2 + b_1 + c_1 \ln r + d_1 r^{-2}) \cos n\varphi + $$
$$+ \sum_{n=2}^{\infty} (a_n r^{n+1} + b_n r^{n-1} + c_n r^{-n+1} + d_n r^{-n-1}) \cos n\varphi \tag{3.2}$$

and analogous expressions for $v_\varphi^{(1)}$ and $p^{(1)}$, containing the arbitrary constants $a_n$, $b_n$, $c_n$, and $d_n$. A partial solution is found from the requirement of the satisfaction of the boundary conditions at the lateral surfaces of the jet, which, after linearization taking account of the smallness of all the quantities depending on the angle $\varphi$, have the form

$$\left[-p^{(1)} + 2\eta \frac{\partial v_r^{(1)}}{\partial r}\right]\Bigg|_{r=r_i^{(0)}} = \frac{(-1)^i \sigma}{(r_i^{(0)})^2}\left[r_i^{(1)} + \frac{d^2 r_i^{(1)}}{d\varphi^2}\right] - 2\eta \frac{\partial^2 v_r^{(0)}}{\partial r^2}\Bigg|_{r=r_i^{(0)}} r_i^{(1)}, \qquad i = 1, 2; \tag{3.3}$$

$$\eta\left[r \frac{\partial}{\partial r}\left(\frac{v_\varphi^{(1)}}{r}\right) + \frac{1}{r}\frac{\partial v_r^{(1)}}{\partial \varphi}\right]\Bigg|_{r=r_i^{(0)}} = \frac{1}{r_i^{(0)}} \frac{dr_i^{(1)}}{d\varphi}\left[p_i + \frac{(-1)^i \sigma}{r} - p^{(0)} + 2\eta \frac{v_r^{(0)}}{r}\right]\Bigg|_{r=r_i^{(0)}}, \qquad i = 1, 2 \tag{3.4}$$

(i = 1 corresponds to the internal surface and i = 2, to the external surface). We seek the perturbation of the thickness of the wall $h^{(1)}(z, \varphi)$ and the mean radius of the jet $\bar{r}^{(1)}(z, \varphi)$ in the form

$$h^{(1)}(z, \varphi) = h_0^{(0)} \sum_{n=1}^{\infty} \alpha_n[s(z)] \cos n\varphi; \quad \bar{r}^{(1)}(z, \varphi) = \bar{r}_0^{(0)} \sum_{n=1}^{\infty} \beta_n[s(z)] \cos n\varphi. \tag{3.5}$$

For the values of $r_i^{(1)}(z, \varphi)$, from formulas (2.1) and (2.2) it follows

$$r_i^{(1)}(z, \varphi) = \bar{r}_0^{(0)} \sum_{n=1}^{\infty} \{(-1)^i \alpha_n[s(z)] \cdot \varepsilon/2 + \beta_n[s(z)]\} \cos n\varphi, \; i = 1, 2, \tag{3.6}$$

where

$$\varepsilon = h_0^{(0)}/\bar{r}_0^{(0)}.$$

Substituting formulas (3.2) and (3.6) and the values of the zero approximation $p^{(0)}$, $v_r^{(0)}$ from [1] into (3.3), (3.4), for each value of n we obtain a system of four linear algebriac equations with respect to $a_n$, $b_n$, $c_n$, $d_n$, containing $\alpha_n$ and $\beta_n$ in the right-hand side. Determining the constants $a_n$, $b_n$, $c_n$, $d_n$ from this system and substituting them into (3.2), we find expressions for the Fourier coefficients $\vartheta_n$ of the perturbation of the radial velocity $v_r^{(1)}$ in terms of the Fourier coefficients $\alpha_n$ and $\beta_n$ of the perturbation of the thickness of the wall $h^{(1)}$ and the mean radius $\bar{r}^{(1)}$, which we shall not write out.

To obtain a closed system of equations with respect to the values of $\alpha_n$ and $\beta_n$, a connection must be established between the change in the perturbations of the internal and external boundaries along z and the perturbation of the radial velocity. To this end, the equation

$$dr_i/dt = v_r|_{r=r_i}, \quad i = 1, 2,$$

connecting the total change with time of the radius of the boundary with the radial velocity, is expanded in terms of the perturbation

$$dr_i^{(0)}/dt = v_r^{(0)}\Big|_{r=r_i^{(0)}}; \quad dr_i^{(1)}/dt = \left[v_r^{(1)} + r_i^{(1)}\partial v_r^{(0)}/\partial r\right]\Big|_{r=r_i^{(0)}}, \quad i = 1, 2. \tag{3.7}$$

Substituting $v_r^{(1)}$ and (3.6) into (3.7) and taking into consideration that

$$dt = dz/v_z^{(0)},$$

we obtain a system of differential equations for the values of $\alpha_n$ and $\beta_n$. We write them separately for $n=1$ and for $n\geq 2$, since the Fourier coefficients in (3.2) for $n=1$ and $n\geq 2$ are written differently. Using (2.5)-(2.7), we have

$$\begin{cases} \dfrac{d\alpha_1}{ds} = -\dfrac{1}{2}\dfrac{\alpha_1}{U}\dfrac{dU}{ds}, \\ \dfrac{d\beta_1}{ds} = -\dfrac{1}{2}\dfrac{\beta_1}{U}\dfrac{dU}{ds}; \end{cases} \tag{3.8}$$

$$\begin{cases} \dfrac{d\alpha_n}{ds} = Q\left[-R\dfrac{\alpha_n}{2} + H\beta_n\right] - \dfrac{1}{2}\dfrac{\alpha_n}{U}\dfrac{dU}{ds}, \; n \geq 2; & (3.9) \\ \dfrac{d\beta_n}{ds} = -\dfrac{Q}{2}\dfrac{R}{H}\left[\dfrac{3}{n^2-1}R\alpha_n + 2H\beta_n\right] - \dfrac{1}{\varepsilon^2}\dfrac{6P}{n^2-1}\dfrac{R^4}{H^4}\beta_n - \dfrac{1}{2}\dfrac{\beta_n}{U}\dfrac{dU}{ds}, & (3.10) \end{cases}$$

where R(s), H(s), U(s) are dimensionless quantities of the zero approximation. The coefficients with $\alpha_n$, $\beta_n$ in the right-hand sides of (3.9), proportional to P and Q, are expanded in powers of a small [in accordance with (2.3a)] parameter $\varepsilon$, and only the leading terms are retained. In the right-hand side of (3.10), the first term is retained along with the second, containing $1/\varepsilon^2$, since, specifically, it is possible that $P=0$, $Q>0$.

A comparison of (3.5) and (1.1) gives the initial conditions for $\alpha_n$, $\beta_n$:

$$\alpha_n|_{s=0} = \varkappa_{n,0}; \quad \beta_n|_{s=0} = \rho_{n,0}, \; n = 1, 2, 3, \ldots \tag{3.11}$$

§4. Let $\alpha'_n$, $\beta'_n$ and $\alpha''_n$, $\beta''_n$ be solutions for the initial conditions

$$\alpha'_n|_{s=0} = 1; \quad \beta'_n|_{s=0} = 0; \tag{4.1}$$

$$\alpha''_n|_{s=0} = 0; \quad \beta''_n|_{s=0} = 1. \tag{4.2}$$

Then from (1.4), (1.5), (2.11), (3.6), (3.11), for the sought coefficients of the transition $A_n$, $B_n$, $C_n$, $D_n$ we have

$$\begin{cases} A_n = U_\infty^{1/2}K^{-1}\alpha'_n|_{s=w}, & B_n = U_\infty^{1/2}K^{-1}\alpha''_n|_{s=w}; \\ C_n = U_\infty^{1/2}K\beta'_n|_{s=w}, & D_n = U_\infty^{1/2}K\beta''_n|_{s=w}. \end{cases} \tag{4.3}$$

For $n=1$, from (3.8), (4.1)-(4.3) we find

$$A_1 = K^{-1}(P, Q, w); \; B_1 = C_1 = 0; \; D_1 = K(P, Q, w), \tag{4.4}$$

from which, by virtue of (1.2), (1.5), $\rho_{1,\infty}=0$, which was to be expected. For $n\geq 2$, we first examine the case $P=0$. The fundamental solutions of the system (3.9), (3.10), taking account of (2.8)-(2.10), are

$$\alpha_n = (H/R)^{\lambda_j}U^{-1/2}; \quad \beta_n = (\lambda_j + 1/2)(H/R)^{\lambda_j - 1}U^{-1/2},$$

where

$$\lambda_j = (-1/4)[1 \pm \sqrt{(n^2-25)/(n^2-1)}], \; j = 1, 2$$

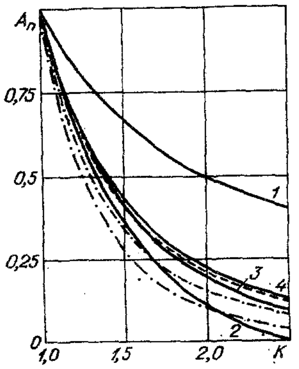


Fig. 3

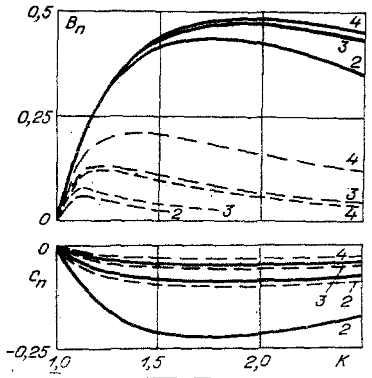


Fig. 4

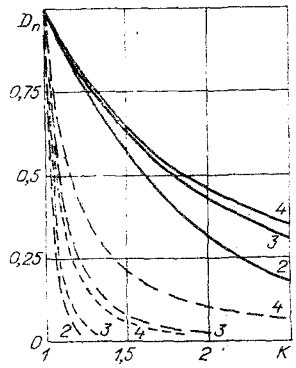


Fig. 5

[for j = 5, there is degeneration: $\lambda_1 = \lambda_2$ and the second solution contains the factor $\ln(H/R)$]. Combining the fundamental solutions in such a way as to satisfy the initial conditions (4.1) or (4.2), from (4.3), taking account of (2.7), (2.11) for $n \leq 4$ (we limit ourselves here to this case), we obtain

$$\begin{cases} A_n = K^{-3/2} \sin\left(\nu_n - \frac{1}{2} \operatorname{tg} \nu_n \cdot \ln K\right) \Big/ \sin \nu_n; \\ B_n = 4K^{-3/2} \sin\left(\frac{1}{2} \operatorname{tg} \nu_n \cdot \ln K\right) \operatorname{ctg} \nu_n; \\ C_n = -\frac{1}{16} B_n / \cos^2 \nu_n, \quad D_n = K^{-3/2} \sin\left(\nu_n + \frac{1}{2} \operatorname{tg} \nu_n \cdot \ln K\right) \Big/ \sin \nu_n, \end{cases}$$

where

$$K = K(O,\ Q,\ w);\ \nu_n = \arcsin\,[\sqrt{(25 - n^2)/24}].$$

For $P > 0$, Eqs. (3.9) and (3.10) cannot be solved in quadratures. The sought values of $A_n$, $B_n$, $C_n$, $D_n$ are found from (4.3), after numerical integration of (3.9), (3.10) for the initial conditions (4.1) or (4.2) for different values of the parameters P, Q, w.

§5. Each of the coefficients is a function of the three variables P, Q, w; here, as in [1], the explicit dependence on w in the range of interest to us $w = \ln(10^3)\ldots\ln(10^5)$ can be neglected.

Figure 3 shows the dependence of the coefficients $A_n$ ($n = 1, 2, 3, 4$) on K with different values of the ratio $P/Q = \Delta p/(2\sigma/\bar{r}_0^{(0)})$, which describes the relationship between the pressure drop of the air and the pressure of the forces of surface tension. (The solid line represents $P/Q = 0$; the dashed line represents $P/Q = 0.2$; the short dashed-dot line represents $P/Q = 0.75$; the long dashed-dot line represents $P/Q = 1$. The numbers on the curves are the numbers of the harmonic n. The unnumbered curves with $P/Q \geq 0.2$ correspond simultaneously to $n = 2, 3, 4$.) The reason for using K(P, Q, w) as an argument, and not Q, is the following: With the condition (2.12), there is always a deviation from similarity [1]; the parameters of the process must be selected in such a way that the coefficient of this deviation will not be too great:

$$[(h_\infty^{(0)}/\bar{r}_\infty^{(0)})/(h_0^{(0)}/\bar{r}_0^{(0)})]^{1/2} \equiv K(P, Q, w) \leqslant K_* \equiv [(h_\infty^{(0)}/\bar{r}_\infty^{(0)})/(h_0^{(0)}/\bar{r}_0^{(0)})_{\min}]^{1/2}, \tag{5.1}$$

since the ratio $h^{(0)}/\bar{r}^{(0)}$ is fixed, and the ratio $h_0^{(0)}/\bar{r}_0^{(0)}$ for the billet cannot be an arbitrarily small quantity; for the typical values of all the parameters $K_* \sim 2$. The limitation on Q flowing out of (5.1) depends on P; therefore, it is not suitable to use Q as an argument.

Figures 4 and 5 give curves for the coefficients $B_n$, $C_n$, $D_n$ ($n = 2, 3, 4$); the long dashes represent $P/Q = 0.1$. There are no curves for $n = 1$, since $B_1 = C_1 = 0$, and $D_1$ is of no interest, by virtue of (1.2). The curves in Figs. 3-5 were plotted for $\varepsilon = 0.1$; curves for $\varepsilon = 0.05$ and 0.2 do not differ significantly from those given.

§6. Amplitude inhomogeneities (nonaxisymmetry of the transverse cross section) of the billet, described by the fact that, for some numbers n, $\varkappa_{n,0} \neq 0$ or $\rho_{n,0} \neq 0$, according to (1.5) generate azimuthal inhomogeneities of $\varkappa_{n,\infty}$ and $\rho_{n,\infty}$ for the microcapillary pulled. The effect of nonaxisymmetry of the billet on the microcapillary is described by the coefficients of the transition $A_n$, $B_n$, $C_n$, $D_n$; the smaller the values of the coefficients, the more strongly are the amplitude inhomogeneities smoothed out during the pulling process. We note that, in a real billet $|h_0^{(0)}\varkappa_{n,0}| \sim |\bar{r}_0^{(0)}\rho_{n,0}|$ for $n \geq 2$; therefore,

$$|\rho_{n,0}| \sim \varepsilon |\varkappa_{n,0}|; \tag{6.1}$$

this circumstance must be taken into account with a pairwise comparison of $A_n$ and $B_n$, $C_n$ and $D_n$.

An electrodynamic calculation [2], confirmed by experiment, shows that the losses with the propagation of light along a lightguide of the type under discussion depend mainly on amplitude inhomogeneities of the thickness of the walls; therefore, we limit ourselves here to investigation of the dependence of the coefficients $A_n$ and $B_n$ on the parameters of the process.

An analysis of the curves for $A_n$, $B_n$ shows that, taking account of (6.1) for $n \geq 2$ and $P/Q = 0$ in formula (1.5a), both terms are of the same order of magnitude. Therefore, in the absence of a pressure drop ($\Delta p = 0$), azimuthal inhomogeneities of the thickness of the wall and the radius of the billet have an approximately equal effect on the aximuthal inhomogeneities of the thickness of the wall of the microcapillary.

For $n \geq 2$ and $P/Q \geq 0.2$, the value of $B_n$ is small in comparison with unity and, taking account of (6.1), in formula (1.5a), the second term can be dropped:

$$\varkappa_{n,\infty} \approx A_n \varkappa_{n,0};$$

by virtue of (4.4) this formula is valid also for $n = 1$ for an arbitrary value of $P/Q$. From the curve for $A_n$ it can be seen that, for $P/Q \geq 0.2$, the value of $A_n$ for $n \geq 2$ does not depend on n. The values of $A_2$, $A_3$, $A_4$ for $K \leqslant 1.7$ are less, the greater the value of $P/Q$; by virtue of (4.4), the value of $A_1$ does not depend on $P/Q$.

Thus, it follows from Figs. 3-5 that, with pulling, there is a decrease in the relative amplitudes of the harmonics of the thickness of the wall and the mean radius. For the first harmonic (the internal and external surfaces of the billet are noncoaxial round cylinders), the decrease is almost wholly determined by the value of the coefficient of the deviation from similarity; therefore, taking account of limitation (5.1), not more than a two-fold decrease in the relative amplitude of the first harmonic of the thickness of the wall is really permissible. For the second (the surfaces of the billet are elliptical cylinders) and succeeding harmonics with the same values of K, a decrease in the relative amplitudes by 5-10 times is permissible. Conditions with $P/Q \sim 1$ are preferable, where in the first place, the azimuthal inhomogeneities of the mean radius of the billet have no effect on the thickness of the wall of the microcapillary and, in the second place, azimuthal inhomogeneities of the thickness of the wall of the billet are more strongly smoothed-out with pulling than with $P/Q = 0$ and the same value of the coefficient of deviation from similarity K.

An analysis of the assumptions made in obtaining Eqs. (3.1) shows that, with values of the parameters P and Q not exceeding a few units, all the significant (not too small in comparison with unit) coefficients $A_n$, $B_n$, $C_n$, $D_n$ correspond to the original statement of the problem.

In the solution of the problem it was assumed that the parameters of the billet are constant along its length. The results, however, can easily be developed for the case where the azimuthal inhomogeneities vary along the length of the billet, if the characteristic length L of such changes is sufficiently great, $L \gg l/w$.

The curves obtained for the coefficients of the transition were used to evaluate the allowance for the azimuthal inhomogeneities of a billet from which, after pulling, a microcapillary is obtained with a given allowance for the value of the deviation of its transverse cross section from a concentric round ring.

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